

NORMALIZATION TECHNIQUES

Integrated-mixed normalization techniques

Notation/Technique	Formula
N22 Ideal Linear Normalization	$n_{ij} = \begin{cases} \min_i x_{ij} & \text{for cost criteria} \\ \frac{x_{ij}}{\max_i x_{ij}} & \text{for benefit criteria} \\ \frac{\min\{x_{ij}, R_j\}}{\max\{x_{ij}, R_j\}} & \text{for reference-based criteria} \end{cases}$
N23 Enhanced Accuracy Normalization	$n_{ij} = \begin{cases} 1 - \frac{\max_i x_{ij} - x_{ij}}{\sum_{i=1}^m (\max_i x_{ij} - x_{ij})} & \text{for benefit criteria} \\ 1 - \frac{x_{ij} - \min_i x_{ij}}{\sum_{i=1}^m (x_{ij} - \min_i x_{ij})} & \text{for cost criteria} \\ \frac{x_{ij} - (\mu + 1, 96\sigma)}{\sum_{i=1}^m (x_{ij} - (\mu + 1, 96\sigma))} & \text{for criteria having normal distribution, if } x_{ij} > (\mu + 1, 96\sigma) \\ \frac{(\mu - 1, 96\sigma) - x_{ij}}{\sum_{i=1}^m ((\mu - 1, 96\sigma) - x_{ij})} & \text{for criteria having normal distribution, if } x_{ij} < (\mu - 1, 96\sigma) \\ 1 & \text{for criteria having normal distribution, if } x_{ij} \in (\mu \pm 1, 96\sigma) \end{cases}$

Normalization techniques independent of the optimization orientation

Technique	All criteria
Z-Score (Non-Monotonic) Normalization	$n_{ij} = e^{-\frac{(x_{ij} - R_j)^2}{2\sigma_j^2}}$
Comprehensive Normalization	$n_{ij} = 1 - e^{-\frac{\ x_{ij} - R_j\ }{(\max\{\max_i x_{ij}, R_j\} - \min\{\min_i x_{ij}, R_j\})}}$
Normalization Equalizing the Average to 1	$n_{ij} = \frac{x_{ij}}{\mu_j}$
Normalization Equalizing Standard Deviation to 1	$n_{ij} = \frac{x_{ij}}{\sigma_j}$
Decimal Normalization	$n_{ij} = \frac{x_{ij}}{10^\rho}, \quad \rho \geq 0$
Reference Based Normalization	$n_{ij} = 1 - \frac{\ x_{ij} - R_j\ }{\max\{\max_i x_{ij}, R_j\} - \min\{\min_i x_{ij}, R_j\}}$
Wu's Reference Based Normalization	$n_{ij} = \frac{\ x_{ij} - R_j\ }{\max_i x_{ij} - R_j}$
Aytekin's Reference Based Normalization	$n_{ij} = 1 - \frac{\ x_{ij} - R_j\ }{\ R_j\ + 10^\rho}, \quad \rho \geq 0$
Range Normalization Between -1 and +1	$n_{ij} = \frac{x_{ij} - \frac{\max_i x_{ij} + \min_i x_{ij}}{2}}{\frac{\max_i x_{ij} - \min_i x_{ij}}{2}}$
Range Normalization Between 0 and +1	$n_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}$
(Zero-) Centering - Mean Normalization	$(\mu)_j = \frac{1}{m} \sum_{i=1}^m x_j^{(i)}$
(Scaling to) Unit Length Normalization	$x' = \frac{x}{\ x\ }$

Normalization techniques depending on the optimization orientation

Techniques	Benefit Criteria	Cost Criteria
Sum-Based Linear Normalization	$n_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}$	$n_{ij} = \frac{1/x_{ij}}{\sum_{i=1}^m 1/x_{ij}}$
Vector Normalization	$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$	$n_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$
Logarithmic Normalization	$n_{ij} = \frac{\ln x_{ij}}{\ln(\prod_{i=1}^m x_{ij})}$	$n_{ij} = \frac{1 - \frac{\ln x_{ij}}{\ln(\prod_{i=1}^m x_{ij})}}{m - 1}$
Maximum-Linear Normalization	$n_{ij} = \frac{x_{ij}}{\max_i x_{ij}}$	$n_{ij} = 1 - \frac{x_{ij}}{\max_i x_{ij}}$

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Sum-Based Linear Normalization	$n_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}$	$n_{ij} = \frac{1/x_{ij}}{\sum_{i=1}^m 1/x_{ij}}$
Vector Normalization	$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$	$n_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$
Logarithmic Normalization	$n_{ij} = \frac{\ln x_{ij}}{\ln(\prod_{i=1}^m x_{ij})}$	$n_{ij} = \frac{1 - \frac{\ln x_{ij}}{\ln(\prod_{i=1}^m x_{ij})}}{m-1}$
Maximum-Linear Normalization	$n_{ij} = \frac{x_{ij}}{\max_i x_{ij}}$	$n_{ij} = 1 - \frac{x_{ij}}{\max_i x_{ij}}$
Minimum-Linear Normalization	$n_{ij} = 1 - \frac{\min_i x_{ij}}{x_{ij}}$	$n_{ij} = \frac{\min_i x_{ij}}{x_{ij}}$
0-1 Interval Normalization Using Max-Min	$n_{ij} = \frac{x_{ij}}{\max_i x_{ij}}$	$n_{ij} = \frac{\min_i x_{ij}}{x_{ij}}$
Jüttler-Körth Normalization	$n_{ij} = 1 - \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij}}$	$n_{ij} = 1 - \frac{\min_i x_{ij} - x_{ij}}{\max_i x_{ij}}$
Stopp Normalization	$n_{ij} = \frac{100x_{ij}}{\max_i x_{ij}}$	$n_{ij} = \frac{100 \min_i x_{ij}}{x_{ij}}$
Nonlinear (Peldschus) Normalization	$n_{ij} = \left(\frac{x_{ij}}{\max_i x_{ij}} \right)^2$	$n_{ij} = \left(\frac{x_{ij}}{\max_i x_{ij}} \right)^3$
Weitendorf's Linear Normalization	$n_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}$	$n_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}$
Z Score Normalization Depending Optimization Aspect	$n_{ij} = \frac{x_{ij} - \mu_j}{\sigma_j} = \frac{x_{ij} - \frac{\sum_{i=1}^m x_{ij}}{m}}{\sqrt{\frac{\sum_{i=1}^m (x_{ij} - \mu_j)^2}{m}}}$	$n_{ij} = -\frac{x_{ij} - \mu_j}{\sigma_j}$

Sources:

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